Coordinatization of Vectors

1. Ordered Basis:

The vector [2, 5] in R^2 can be expressed in terms of the standard basis vectors as $2e_1 + 5e_2$. In a non-zero vector space V there are no order for the basis. This is because set notation does not denote order. $\{b_1, b_2\} = \{b_2, b_1\}$. However, to describe order, we use (,). E.g. (b_1, b_2). We denoted an ordered basis of n vectors in V by $B = (b_1, b_2, \ldots b_n)$.

2. Coordinatization of Basis:

Let V be a finite-dimensional vector space and let $B = (b_1, b_2, \ldots b_n)$ be a basis for V. Every vector v in V can be expressed in the form $v = r_1b_1 + r_2b_2 + \ldots r_nb_n$, for unique scalars $r_1, r_2, \ldots r_n$. The vector $[r_1, r_2, \ldots r_n]$ in R^n with v.

3. Coordinate Vector Relative to an Ordered Basis:

Let $B = (b_1, b_2, \ldots b_n)$ be an ordered basis for a finite dimensional vector space V, and let $v = r_1b_1 + r_2b_2 + \ldots r_nb_n$. The vector $[r_1, r_2, \ldots r_n]$ is the coordinate vector of v relative to the ordered basis B, and is denoted by v_B.

E.g. 1. Find the coordinate vectors of $[1,-1]$ and of $[-1,-8]$ relative to the ordered basis $B = ([1,-1], [1,2])$ of R^2.

Solution:

$$[1,-1] = r_1 [1,-1] + r_2 [1,2]$$
$$= 1 [1,-1] + 0 [1,2]$$

$$\therefore [1,-1]_B = [1,0]$$

$$[-1,-8] = r_1 [1,-1] + r_2 [1,2]$$

$$-1 = r_1 + r_2$$
$$-8 = -r_1 + 2r_2$$

$$r_1 = 2, \; r_2 = -3$$

$$\therefore [-1, 8]_B = [2,-3]$$

E.g. 2 Find the coordinate vector of $[1,2,-2]$ relative to the ordered basis $B = ([1,1,1], [1,2,0], [1,0,1])$.

Solution:

$$\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = r_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + r_2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + r_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$r_1 + r_2 + r_3 = 1$
$r_1 + 2r_2 + 0 = 2$
$r_1 + 0 + r_3 = -2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 2 \\ 1 & 0 & 1 & -2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\therefore \ [1,2,-2]_B = [-4,3,2]$

The procedure to find the coordinate vector of v in R^n relative to an ordered basis $B = (b_1, b_2, \dots b_n)$:

Step 1: Write vectors as column vectors in the form

$$\left[\begin{array}{cccc|c} | & | & & | & | \\ b_1 & b_2 & \cdots & b_n & v \\ | & | & & | & | \end{array}\right]$$

Step 2: RREF the matrix to get $[I \mid v_B]$ where I is an $n \times n$ matrix.